

14.3

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The formula for aspartame is  $C_{14}H_{18}N_2O_5$ . From the periodic table in the back of the book, we obtain the atomic weights:

C	12.011 a.m.u.
H	1.00794 a.m.u.
N	14.0067 a.m.u.
O	15.9994 a.m.u.

a. Therefore the formula weight (in atomic mass units) for aspartame is

$$14(12.011 \text{ a.m.u.}) + 18(1.00794 \text{ a.m.u.}) + 2(14.0067 \text{ a.m.u.}) + 5(15.9994 \text{ a.m.u.}) =$$

294.307 a.m.u.

b. So 294.307 g of aspartame equals one mole. And one mole of anything is comprised of  $6.022 \times 10^{23}$  molecules (Avogadro's number). This means

$$294.307 \text{ g of aspartame} = 6.022 \times 10^{23} \text{ molecules}$$

Therefore the mass  $m$  of a single molecule of aspartame is

$$m = \frac{294.307 \text{ g}}{6.022 \times 10^{23}} = 4.887 \times 10^{-22} \text{ g} = 4.887 \times 10^{-25} \text{ kg}$$

14.12

$$P_1 = 65.0 \text{ atm.}$$

$$T_1 = 288 \text{ K}$$

$$P_2 = 1.00 \text{ atm.}$$

$$T_2 = 297 \text{ K}$$

$$V_1 = 1.00 \text{ m}^3$$

$$V_2 = ?$$

$$\text{In room \# 1} \quad P_1 V_1 = n_1 R T_1 \quad \# 1$$

$$\text{In room \# 2} \quad P_2 V_2 = n_2 R T_2 \quad \# 2$$

Dividing eq # 2 by eq # 1 and assuming  $n_1 = n_2$  (i.e. no gas escapes)

$$\frac{P_2 V_2}{P_1 V_1} = \frac{n_2 T_2}{n_1 T_1} = \frac{T_2}{T_1} \quad \# 3$$

Solving for  $V_2$  in eq # 3, we obtain

$$V_2 = V_1 \left( \frac{P_1}{P_2} \right) \left( \frac{T_2}{T_1} \right) = 1.00 \text{ m}^3 \left( \frac{65.0 \text{ atm}}{1.00 \text{ atm}} \right) \left( \frac{297 \text{ K}}{288 \text{ K}} \right) = \boxed{67.0 \text{ m}^3}$$

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For an ideal gas,  $PV = nRT$

$$\therefore n = \frac{PV}{RT} = \frac{(1.1 \times 10^5 \text{ N/m}^2)(5400 \text{ m}^3)}{(8.314 \text{ J/mole K})(280 \text{ K})} = 255,163 \text{ moles.}$$

The formula weight for monatomic helium is 4.00260 (from the periodic table of the elements in the back of our textbook), which means that

$$1 \text{ mole He} = 4.00260 \text{ g}$$

$$\therefore m = (255,163 \text{ moles}) \left( \frac{4.00260 \text{ g}}{1 \text{ mole}} \right) = 1,021,317 \text{ g} = 1,021.317 \text{ kg}$$

Rounded to two significant figures,  $m = \boxed{1000 \text{ kg}}$

14.15

$$T_1 = 305 \text{ K}$$

$$V_2 = \frac{V_1}{16}$$

$$P_2 = 48.5 P_1$$

$$T_2 = ?$$

Before compression  $P_1 V_1 = n_1 R T_1$  #1

After compression  $P_2 V_2 = n_2 R T_2$  #2

Dividing eq #2 by eq #1, we have

$$\frac{P_2 V_2}{P_1 V_1} = \frac{n_2 R T_2}{n_1 R T_1} = \frac{n_2 T_2}{n_1 T_1} \quad \#3$$

Assuming that no molecules are gained or lost (i.e.  $n_1 = n_2$ ), eq #3 simplifies to

$$\frac{P_2 V_2}{P_1 V_1} = \frac{T_2}{T_1} \quad \#4$$

and we can solve for  $T_2$ :

$$T_2 = T_1 \left( \frac{P_2 V_2}{P_1 V_1} \right) = 305 \text{ K} \left[ \frac{(48.5 P_1) \left( \frac{V_1}{16} \right)}{P_1 V_1} \right] = 305 \text{ K} \left[ (48.5) \left( \frac{1}{16} \right) \right] = \boxed{925 \text{ K}}$$

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$$v_{\text{RMS}} = 650 \text{ m/s.}$$

$$T = ?$$

From the kinetic theory of gases

$$\frac{1}{2} m \overline{v^2} = \frac{3}{2} kT \rightarrow T = \frac{m \overline{v^2}}{3k} \quad \#1.$$

To use eq #1, we will need to first determine the mass  $m$  of a single molecule of carbon dioxide ( $\text{CO}_2$ ). The formula weight of  $\text{CO}_2$  is

$$12 + 2(16) = 44$$

Therefore 1 mole  $\text{CO}_2 = 44 \text{ g} = 6.022 \times 10^{23}$  molecules.  
and therefore the mass of one molecule of  $\text{CO}_2$  is

$$m = \frac{44 \text{ g}}{6.022 \times 10^{23}} = 7.307 \times 10^{-23} \text{ g} = 7.307 \times 10^{-26} \text{ kg}$$

Notice that the mass is reexpressed in kilograms to be consistent with the units of the Boltzmann constant.

Now substituting into eq #1,

$$T = \frac{(7.307 \times 10^{-26} \text{ kg})(650 \text{ m/s})^2}{3(1.38 \times 10^{-23} \text{ J/K})} = \boxed{746 \text{ K}}$$

14.30

From the kinetic theory of gases,

$$\frac{1}{2} m \overline{v^2} = \frac{3}{2} kT \quad \#1$$

Because the water and the carbon dioxide molecules are at the same temperatures, they must have the same average kinetic energies. So,

$$\frac{1}{2} m_{\text{H}_2\text{O}} \overline{v_{\text{H}_2\text{O}}^2} = \frac{1}{2} m_{\text{CO}_2} \overline{v_{\text{CO}_2}^2} \quad \#2$$

from which we obtain

$$\overline{v_{\text{CO}_2}^2} = \overline{v_{\text{H}_2\text{O}}^2} \left( \frac{m_{\text{H}_2\text{O}}}{m_{\text{CO}_2}} \right) \quad \#3$$

Taking the square root of this equation yields

$$v_{\text{RMS CO}_2} = v_{\text{RMS H}_2\text{O}} \sqrt{\frac{m_{\text{H}_2\text{O}}}{m_{\text{CO}_2}}} \quad \#4$$

The "m" in the equations above represent the mass of a single molecule, found by dividing the formula weight (f.w.) by Avogadro's number,  $N_A$ . So,

$$\frac{m_{\text{H}_2\text{O}}}{m_{\text{CO}_2}} = \frac{\text{f.w. H}_2\text{O} / N_A}{\text{f.w. CO}_2 / N_A} = \frac{\text{f.w. H}_2\text{O}}{\text{f.w. CO}_2}$$

Since  $\text{f.w. H}_2\text{O} = 2(1) + 16 = 18$  and  $\text{f.w. CO}_2 = 12 + 2(16) = 44$

$$\frac{m_{\text{H}_2\text{O}}}{m_{\text{CO}_2}} = \frac{18}{44} \quad \#5$$

Substituting eq 5 into eq 4 along with  $v_{\text{RMS H}_2\text{O}} = 648 \text{ m/s}$

$$v_{\text{RMS CO}_2} = 648 \text{ m/s} \sqrt{\frac{18}{44}} = 414.462 \text{ m/s}$$

Rounded to two significant figures,  $v_{\text{RMS CO}_2} = \boxed{410 \text{ m/s}}$

14.54

First, we can determine the temperature of the  $\text{SO}_2$  by using the equation of state for an ideal gas

$$PV = nRT \quad \#1$$

$$T = \frac{PV}{nR} = \frac{(2.12 \times 10^4 \text{ N/m}^2)(50.0 \text{ m}^3)}{(4.21 \text{ moles})(8.314 \text{ J/mole K})} = 302.84 \text{ K}$$

Next, from the kinetic theory of gases equation,  $\frac{1}{2} m \bar{v}^2 = \frac{3}{2} kT$ , we can solve for the rms speed

$$v_{\text{RMS}} = \sqrt{\bar{v}^2} = \sqrt{\frac{3kT}{m}} \quad \#2$$

The "m" in eq 2 represents the mass of a  $\text{SO}_2$  molecule and is found by dividing the formula weight (f.w.) expressed in grams by Avogadro's number ( $N_A$ ). Since the f.w. for  $\text{SO}_2$  equals the atomic weight of one sulfur and two oxygen atoms

$$\text{f.w.}_{\text{SO}_2} = 32 + 2(16) = 64$$

So, 64 g of  $\text{SO}_2 = 6.02 \times 10^{23}$  molecules and therefore

$$m = \frac{64 \text{ g/mole}}{6.02 \times 10^{23} \text{ molecules/mole}} = 1.0631 \times 10^{-22} \text{ g} = 1.0631 \times 10^{-25} \text{ kg}$$

Substituting this value into eq 2 along with the temperature and the Boltzmann constant  $k = 1.38 \times 10^{-23} \text{ J/K}$

$$v_{\text{RMS}} = \sqrt{\frac{3(1.38 \times 10^{-23} \text{ J/K})(302.84 \text{ K})}{1.0631 \times 10^{-25} \text{ kg}}} = \boxed{343 \text{ m/s}}$$