CHAPTER 1
FUNDAMENTAL CONCEPTS: VECTORS

1.1 (a) $\vec{A}+\vec{B}=(\hat{i}+\hat{j})+(\hat{j}+\hat{k})=\hat{i}+2\hat{j}+\hat{k}$

$$\left|\vec{A}+\vec{B}\right|=(1+4+1)^{\frac{1}{2}}=\sqrt{6}$$

(b) $3\vec{A}-2\vec{B}=3(\hat{i}+\hat{j})-2(\hat{j}+\hat{k})=3\hat{i}+\hat{j}-2\hat{k}$

(c) $\vec{A}\cdot\vec{B}=(1)(0)+(1)(1)+(0)(1)=1$

(d) $$\vec{A}\times\vec{B}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 0\\ 0 & 1 & 1\end{vmatrix}=\hat{i}(1-0)+\hat{j}(0-1)+\hat{k}(1-0)=\hat{i}-\hat{j}+\hat{k}$$

$$\left|\vec{A}\times\vec{B}\right|=(1+1+1)^{\frac{1}{2}}=\sqrt{3}$$

1.2 (a) $\vec{A}\cdot\left(\vec{B}+\vec{C}\right)=\left(2\hat{i}+\hat{j}\right)\cdot\left(\hat{i}+4\hat{j}+\hat{k}\right)=(2)(1)+(1)(4)+(0)(1)=6$

$$\left(\vec{A}+\vec{B}\right)\cdot\vec{C}=\left(3\hat{i}+\hat{j}+\hat{k}\right)\cdot 4\hat{j}=(3)(0)+(1)(4)+(1)(0)=4$$

(b) $$\vec{A}\cdot\left(\vec{B}\times\vec{C}\right)=\begin{vmatrix}2 & 1 & 0\\ 1 & 0 & 1\\ 0 & 4 & 0\end{vmatrix}=-8$$

$$\left(\vec{A}\times\vec{B}\right)\cdot\vec{C}=\vec{A}\cdot\left(\vec{B}\times\vec{C}\right)=-8$$

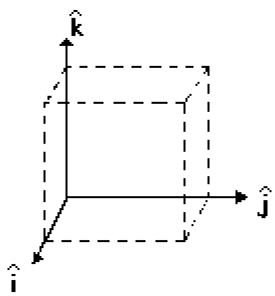
(c) $\vec{A}\times\left(\vec{B}\times\vec{C}\right)=\left(\vec{A}\cdot\vec{C}\right)\vec{B}-\left(\vec{A}\cdot\vec{B}\right)\vec{C}=4\left(\hat{i}+\hat{k}\right)-2\left(4\hat{j}\right)=4\hat{i}-8\hat{j}+4\hat{k}$

$$\left(\vec{A}\times\vec{B}\right)\times\vec{C}=-\vec{C}\times\left(\vec{A}\times\vec{B}\right)=-\left[\left(\vec{C}\cdot\vec{B}\right)\vec{A}-\left(\vec{C}\cdot\vec{A}\right)\vec{B}\right]$$
$$=-\left[0\left(2\hat{i}+\hat{j}\right)-4\left(\hat{i}+\hat{k}\right)\right]=4\hat{i}+4\hat{k}$$

1.3 $\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB} = \frac{(a)(a)+(2a)(2a)+(0)(3a)}{\sqrt{5a^2}\sqrt{14a^2}} = \frac{5a^2}{a^2\sqrt{5}\sqrt{14}}$

$$\theta = \cos^{-1}\sqrt{\frac{5}{14}} \approx 53°$$

1.4



(a) $\vec{A} = \hat{i} + \hat{j} + \hat{k}$: *body diagonal*

$$A = \left|\vec{A}\cdot\vec{A}\right| = \sqrt{\hat{i}\cdot\hat{i} + \hat{j}\cdot\hat{j} + \hat{k}\cdot\hat{k}} = \sqrt{3}$$

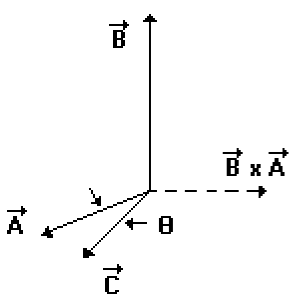
(b) $\vec{B} = \hat{i} + \hat{j}$: *face diagonal*

$$B = \left|\vec{B}\cdot\vec{B}\right| = \sqrt{2}$$

(c) $\vec{C} = \vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix}$

(d) $\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB} = \frac{1-1}{\sqrt{3}\sqrt{2}} = 0 \qquad \therefore \theta = 90^\circ$

1.5



$B = \left|\vec{B}\right| = \left|\vec{A}\times\vec{C}\right| = AC\sin\theta \qquad \therefore C_y = C\sin\theta = \frac{B}{A}$

$\vec{A}\cdot\vec{C} = AC\cos\theta = u \qquad \therefore C_x = C\cos\theta = \frac{u}{A}$

$$\vec{C} = \frac{\vec{A}}{A}C_x + \frac{\vec{B}\times\vec{A}}{\left|\vec{B}\times\vec{A}\right|}C_y = \frac{u}{A^2}\vec{A} + \frac{\vec{B}\times\vec{A}}{AB}\left(\frac{B}{A}\right)$$

$$= \frac{u}{A^2}\vec{A} + \frac{1}{A^2}\vec{B}\times\vec{A}$$

1.6 $\frac{d\vec{A}}{dt} = \hat{i}\frac{d}{dt}(\alpha t) + \hat{j}\frac{d}{dt}\left(\beta t^2\right) + \hat{k}\frac{d}{dt}\left(\gamma t^3\right) = \hat{i}\alpha + \hat{j}2\beta t + \hat{k}3\gamma t^2$

$$\frac{d^2\vec{A}}{dt^2} = \hat{j}2\beta + \hat{k}6\gamma t$$

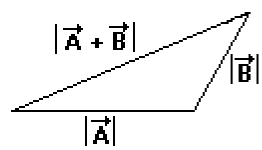
1.7 $0 = \vec{A}\cdot\vec{B} = (q)(q) + (3)(-q) + (1)(2) = q^2 - 3q + 2$

$(q-2)(q-1) = 0$, $\quad q = 1$ or 2

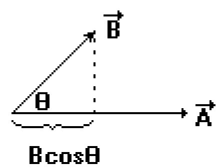
1.8 $\left|\vec{A}+\vec{B}\right|^2 = \left(\vec{A}+\vec{B}\right)\cdot\left(\vec{A}+\vec{B}\right) = A^2 + B^2 + 2\vec{A}\cdot\vec{B}$

$\left[\left|\vec{A}\right| + \left|\vec{B}\right|\right]^2 = A^2 + B^2 + 2AB$

Since $\vec{A}\cdot\vec{B} = AB\cos\theta \le AB$, $\quad \left|\vec{A}+\vec{B}\right| \le \left|\vec{A}\right| + \left|\vec{B}\right|$



$\left|\vec{A}\cdot\vec{B}\right| = \left|AB\cos\theta\right| = \left|\vec{A}\right|\left|\vec{B}\right|\left|\cos\theta\right| \le \left|\vec{A}\right|\left|\vec{B}\right|$



$B\cos\theta \le B$

1.9 Show $\vec{A}\times\left(\vec{B}\times\vec{C}\right) = \left(\vec{A}\cdot\vec{C}\right)\vec{B} - \left(\vec{A}\cdot\vec{B}\right)\vec{C}$

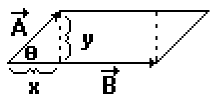
or $$\vec{A}\times\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ B_x & B_y & B_z\\ C_x & C_y & C_z\end{vmatrix} = \left(A_xC_x + A_yC_y + A_zC_z\right)\vec{B} - \left(A_xB_x + A_yB_y + A_zB_z\right)\vec{C}$$

$$\begin{aligned}
&= \left(A_xB_xC_x + A_yB_xC_y + A_zB_xC_z - A_xB_xC_x - A_yB_yC_x - A_zB_zC_x\right)\hat{i}\\
&+ \left(A_xB_yC_x + A_yB_yC_y + A_zB_yC_z - A_xB_xC_y - A_yB_yC_y - A_zB_zC_y\right)\hat{j}\\
&+ \left(A_xB_zC_x + A_yB_zC_y + A_zB_zC_z - A_xB_xC_z - A_yB_yC_z - A_zB_zC_z\right)\hat{k}
\end{aligned}$$

$$\begin{aligned}
&\left(A_yB_xC_y + A_zB_xC_z - A_yB_yC_x - A_zB_zC_x\right)\hat{i}\\
= &+\left(A_xB_yC_x + A_zB_yC_z - A_xB_xC_y - A_zB_zC_y\right)\hat{j}\\
&+\left(A_xB_zC_x + A_yB_zC_y - A_xB_xC_z - A_yB_yC_z\right)\hat{k}
\end{aligned}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_yC_z - B_zC_y & B_zC_x - B_xC_z & B_xC_y - B_yC_x \end{vmatrix} = \begin{matrix} \hat{i}\left(A_yB_xC_y - A_yB_yC_x - A_zB_zC_x + A_zB_xC_z\right) \\ +\hat{j}\left(A_zB_yC_z - A_zB_zC_y - A_xB_xC_y + A_xB_yC_x\right) \\ +\hat{k}\left(A_xB_zC_x - A_xB_xC_z - A_yB_yC_z + A_yB_zC_y\right) \end{matrix}$$

1.10



$$y = A\sin\theta$$

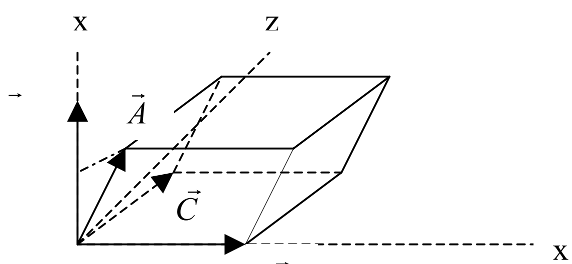
$$\text{A} = 2\left(\frac{1}{2}xy\right) + y\left(B - x\right) = xy + yB - xy = AB\sin\theta$$

$$\text{A} = \left|\vec{A}\times\vec{B}\right|$$

1.11

$$\vec{A}\cdot\left(\vec{B}\times\vec{C}\right) = \vec{A}\cdot\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} = -\begin{vmatrix} B_x & B_y & B_z \\ A_x & A_y & A_z \\ C_x & C_y & C_z \end{vmatrix} = -\vec{B}\cdot\left(\vec{A}\times\vec{C}\right)$$

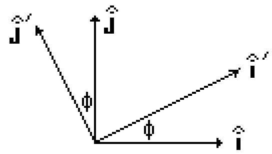
1.12



Let $\vec{A}$ = (Ax,Ay,Az), $\vec{B}$ = (0,By,0) and $\vec{C}$ = (0,Cy,Cz)
C_z is the perpendicular distance between the plane $\vec{A}, \vec{B}$ and its opposite. $\vec{u} = \vec{B}\,x\,\vec{C}$ is directed along the x-axis since the vectors $\vec{B}$, $\vec{C}$ are in the y,z plane. $u_x = \left|\vec{B}\,x\,\vec{C}\right|$ = B_yC_z is the area of the parallelogram formed by the vectors $\vec{B}$, $\vec{C}$. Multiply that area times the height of plane $\vec{A}, \vec{B}$ = A_x to get the volume of the parallopiped

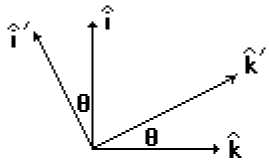
$$V = A_xu_x = A_x\,B_y\,C_z = \vec{A}\bullet\left(\vec{B}\,x\,\vec{C}\right)$$

1.13 For rotation about the z axis:



$$\hat{i}\cdot\hat{i}' = \cos\phi = \hat{j}\cdot\hat{j}', \quad \hat{k}\cdot\hat{k}' = 1$$
$$\hat{i}\cdot\hat{j}' = -\sin\phi$$
$$\hat{j}\cdot\hat{i}' = \sin\phi$$

For rotation about the y' axis:



$$\hat{i}\cdot\hat{i}' = \cos\theta = \hat{k}\cdot\hat{k}', \quad \hat{j}\cdot\hat{j}' = 1$$
$$\hat{i}\cdot\hat{k}' = \sin\theta$$
$$\hat{k}\cdot\hat{i}' = -\sin\theta$$

$$\vec{T} = \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix} \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \\ \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \end{pmatrix}$$

1.14

$$\hat{i}\cdot\hat{i}' = \cos 30^\circ = \frac{\sqrt{3}}{2} \qquad \hat{j}\cdot\hat{i}' = \sin 30^\circ = \frac{1}{2} \qquad \hat{k}\cdot\hat{i}' = 0$$

$$\hat{i}\cdot\hat{j}' = -\sin 30^\circ = -\frac{1}{2} \qquad \hat{j}\cdot\hat{j}' = \cos 30^\circ = \frac{\sqrt{3}}{2} \qquad \hat{k}\cdot\hat{j}' = 0$$

$$\hat{i}\cdot\hat{k}' = 0 \qquad \hat{j}\cdot\hat{k}' = 0 \qquad \hat{k}\cdot\hat{k}' = 1$$

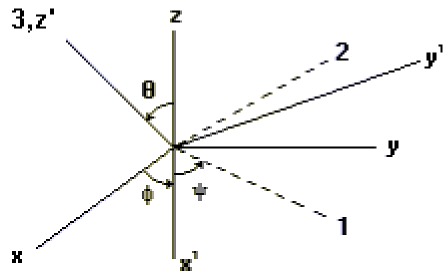
$$\begin{bmatrix} A_{x'} \\ A_{y'} \\ A_{z'} \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} = \begin{bmatrix} \sqrt{3} + \frac{3}{2} \\ \frac{3}{2}\sqrt{3} - 1 \\ -1 \end{bmatrix}$$

$$\vec{A} = 3.232\hat{i}' + 1.598\hat{j}' - \hat{k}'$$

1.15 1. Rotate thru ϕ about z-axis $\qquad \phi = 45^\circ \qquad R_\phi$

2. Rotate thru θ about x'-axis $\qquad \theta = 45^\circ \qquad R_\theta$

3. Rotate thru ψ about z'-axis $\qquad \psi = 45^\circ \qquad R_\psi$



$$R_\phi = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R_\theta = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \qquad R_\psi = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$R(\psi,\theta,\phi) = R_\psi R_\theta R_\phi = \begin{pmatrix} \frac{1}{2}-\frac{1}{2\sqrt{2}} & \frac{1}{2}+\frac{1}{\sqrt{2}} & \frac{1}{2} \\ -\frac{1}{2}-\frac{1}{2\sqrt{2}} & -\frac{1}{2}+\frac{1}{2\sqrt{2}} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} \text{ or } \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = R(\psi,\theta,\phi) \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$$

Condition is: $\vec{x}' = R\vec{x}$ where $\vec{x}' = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $\vec{x} = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$

Since $\vec{x} \cdot \vec{x} = 1$ we have: $\psi^2 + \beta^2 + \alpha^2 = 1$

After a lot of algebra: $\alpha = \frac{1}{2} - \frac{\sqrt{2}}{4}, \ \beta = \frac{1}{2} + \frac{\sqrt{2}}{4}, \ \gamma = \frac{1}{2}$

1.16 $\vec{v} = v\hat{\tau} = ct\hat{\tau}$

$$\vec{a} = \dot{v}\hat{\tau} + \frac{v^2}{\rho}\hat{n} = c\hat{\tau} + \frac{c^2t^2}{b}\hat{n}$$

at $t=\sqrt{\frac{b}{c}}$, $\vec{v}=\hat{\tau}\sqrt{bc}$ and $\vec{a}=c\hat{\tau}+c\hat{n}$

$$\cos\theta=\frac{\vec{v}\cdot\vec{a}}{va}=\frac{c\sqrt{bc}}{\sqrt{bc}\sqrt{2c^2}}=\frac{1}{\sqrt{2}}$$

$$\theta=45^\circ$$

1.17 $\vec{v}(t)=-\hat{i}b\omega\sin(\omega t)+\hat{j}2b\omega\cos(\omega t)$

$$|\vec{v}|=\left(b^2\omega^2\sin^2\omega t+4b^2\omega^2\cos^2\omega t\right)^{\frac{1}{2}}=b\omega\left(1+3\cos^2\omega t\right)^{\frac{1}{2}}$$

$$\vec{a}(t)=-\hat{i}b\omega^2\cos\omega t-\hat{j}2b\omega^2\sin\omega t$$

$$|\vec{a}|=b\omega^2\left(1+3\sin^2\omega t\right)^{\frac{1}{2}}$$

at $t=0$, $|\vec{v}|=2b\omega$; at $t=\frac{\pi}{2\omega}$, $|\vec{v}|=b\omega$

1.18 $\vec{v}(t)=\hat{i}b\omega\cos\omega t-\hat{j}b\omega\sin\omega t+\hat{k}2ct$

$$\vec{a}(t)=-\hat{i}b\omega^2\sin\omega t-\hat{j}b\omega^2\cos\omega t+\hat{k}2c$$

$$|\vec{a}|=\left(b^2\omega^4\sin^2\omega t+b^2\omega^4\cos^2\omega t+4c^2\right)^{\frac{1}{2}}=\left(b^2\omega^4+4c^2\right)^{\frac{1}{2}}$$

1.19 $\vec{v}=\dot{r}\hat{e}_r+r\dot{\theta}\hat{e}_\theta=bke^{kt}\hat{e}_r+bce^{kt}\hat{e}_\theta$

$$\vec{a}=\left(\ddot{r}-r\dot{\theta}^2\right)\hat{e}_r+\left(r\ddot{\theta}+2\dot{r}\dot{\theta}\right)\hat{e}_\theta=b\left(k^2-c^2\right)e^{kt}\hat{e}_r+2bcke^{kt}\hat{e}_\theta$$

$$\cos\phi=\frac{\vec{v}\cdot\vec{a}}{va}=\frac{b^2k\left(k^2-c^2\right)e^{2kt}+2b^2c^2ke^{2kt}}{be^{kt}\left(k^2+c^2\right)^{\frac{1}{2}}be^{kt}\left[\left(k^2-c^2\right)^2+4c^2k^2\right]^{\frac{1}{2}}}$$

$$\cos\phi=\frac{k\left(k^2+c^2\right)}{\left(k^2+c^2\right)^{\frac{1}{2}}\left(k^2+c^2\right)}=\frac{k}{\left(k^2+c^2\right)^{\frac{1}{2}}},\quad \text{a constant}$$

1.20 $\vec{a}=\left(\ddot{R}-R\phi\right)\hat{e}_R+\left(2\dot{R}\phi+R\phi\right)\hat{e}_\phi+\ddot{z}\hat{e}_z$

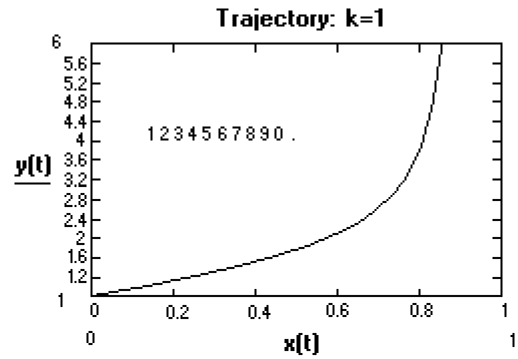
$$\vec{a}=-b\omega^2\hat{e}_R+2c\hat{e}_z$$

$$|\vec{a}|=\left(b^2\omega^4+4c^2\right)^{\frac{1}{2}}$$

1.21 $\vec{r}(t) = \hat{i}\left(1-e^{-kt}\right) + \hat{j}e^{kt}$

$$\vec{r}(t) = \hat{i}ke^{-kt} + \hat{j}ke^{kt}$$

$$\vec{r}(t) = -\hat{i}k^2e^{-kt} + \hat{j}k^2e^{kt}$$



1.22 $\vec{v} = \hat{e}_r\dot{r} + \hat{e}_\phi r\dot{\phi}\sin\theta + \hat{e}_\theta r\dot{\theta}$

$$\vec{v} = \hat{e}_\phi b\omega\sin\left\{\frac{\pi}{2}\left[1+\frac{1}{4}\cos\left(4\omega t\right)\right]\right\} - \hat{e}_\theta b\frac{\pi}{2}\omega\sin\left(4\omega t\right)$$

$$\vec{v} = \hat{e}_\phi b\omega\cos\left[\frac{\pi}{8}\cos\left(4\omega t\right)\right] - \hat{e}_\theta b\omega\frac{\pi}{2}\sin\left(4\omega t\right)$$

$$\left|\vec{v}\right| = b\omega\left[\cos^2\left(\frac{\pi}{8}\cos 4\omega t\right) + \frac{\pi^2}{4}\sin^2 4\omega t\right]^{\frac{1}{2}}$$

Path is sinusoidal oscillation about the equator.

1.23 $\vec{v}\cdot\vec{v} = v^2$

$$\frac{d\vec{v}}{dt}\cdot\vec{v} + \vec{v}\cdot\frac{d\vec{v}}{dt} = 2v\dot{v}$$

$$2\vec{v}\cdot\vec{a} = 2v\dot{v}$$

$$\vec{v}\cdot\vec{a} = v\dot{v}$$

1.24 $\frac{d}{dt}\left[\vec{r}\cdot\left(\vec{v}\times\vec{a}\right)\right]=\frac{d\vec{r}}{dt}\cdot\left(\vec{v}\times\vec{a}\right)+\vec{r}\cdot\frac{d}{dt}\left(\vec{v}\times\vec{a}\right)$

$$=\vec{v}\cdot\left(\vec{v}\times\vec{a}\right)+\vec{r}\cdot\left[\left(\frac{d\vec{v}}{dt}\times\vec{a}\right)+\left(\vec{v}\times\frac{d\vec{a}}{dt}\right)\right]$$

$$=0+\vec{r}\cdot\left[0+\left(\vec{v}\times\dot{\vec{a}}\right)\right]$$

$$\frac{d}{dt}\left[\vec{r}\cdot\left(\vec{v}\times\vec{a}\right)\right]=\vec{r}\cdot\left(\vec{v}\times\dot{\vec{a}}\right)$$

1.25 $\vec{v}=v\hat{\tau}$ and $\vec{a}=a_\tau\hat{\tau}+a_n\hat{n}$

$\vec{v}\cdot\vec{a}=va_\tau$, so $a_\tau=\frac{\vec{v}\cdot\vec{a}}{v}$

$a^2=a_\tau^2+a_n^2$, so $a_n=\left(a^2-a_\tau^2\right)^{\frac{1}{2}}$

1.26 For 1.14, $a_\tau=\frac{-b^2\omega^3\cos\omega t\cdot\sin\omega t+b^2\omega^3\sin\omega t\cdot\cos\omega t+4c^2t}{\left(b^2\omega^2\cos^2\omega t+b^2\omega^2\sin^2\omega t+4c^2t^2\right)^{\frac{1}{2}}}$

$$a_\tau=\frac{4c^2t}{\left(b^2\omega^2+4c^2t^2\right)^{\frac{1}{2}}}$$

$$a_n=\left(b^2\omega^2+4c^2-\frac{16c^4t^2}{b^2\omega^2+4c^2t^2}\right)^{\frac{1}{2}}$$

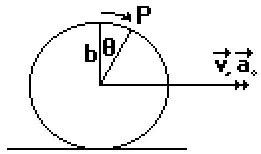
For 1.15, $a_\tau=\frac{b^2k\left(k^2-c^2\right)e^{2kt}+2b^2c^2ke^{2kt}}{be^{kt}\left(k^2+c^2\right)^{\frac{1}{2}}}=bke^{kt}\left(k^2+c^2\right)^{\frac{1}{2}}$

$$a_n=\left[b^2e^{2kt}\left(k^2+c^2\right)^2-b^2k^2e^{2kt}\left(k^2+c^2\right)\right]^{\frac{1}{2}}=bce^{kt}\left(k^2+c^2\right)^{\frac{1}{2}}$$

1.27 $\vec{v}=v\hat{\tau}$, $\vec{a}=\dot{v}\hat{\tau}+\frac{v^2}{\rho}\hat{n}$

$$\left|\vec{v}\times\vec{a}\right|=v\cdot a_n=v\frac{v^2}{\rho}=\frac{v^3}{\rho}$$

1.28



$$\vec{r}_{\circ P} = \hat{i} b \sin\theta + \hat{j} b \cos\theta$$
$$\vec{v}_{rel} = \hat{i} b\dot{\theta} \cos\theta - \hat{j} b\dot{\theta} \sin\theta$$
$$\vec{a}_{rel} = \hat{i} b\left(\ddot{\theta}\cos\theta - \dot{\theta}^2 \sin\theta\right) - \hat{j} b\left(\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta\right)$$

at the point $\theta = \frac{\pi}{2}$, $\vec{v}_{rel} = -\vec{v}$

So, $\left|\vec{v}_{rel}\right| = b\dot{\theta} = v$

$$\dot{\theta} = \frac{v}{b} \qquad \ddot{\theta} = \frac{\dot{v}}{b} = \frac{a_\circ}{b}$$

Now, $\vec{a}_{rel} = \dot{v}_{rel}\hat{\tau} + \frac{v_{rel}^2}{\rho}\hat{n} = a_\circ\hat{\tau} + \frac{v^2}{b}\hat{n}$

$$\left|\vec{a}_{rel}\right| = \left(a_\circ^2 + \frac{v^4}{b^2}\right)^{\frac{1}{2}}$$

$\vec{v}_P = \vec{v} + \vec{v}_{rel}$ and $\vec{a}_P = \vec{a}_\circ + \vec{a}_{rel}$

$$\vec{a}_P = \hat{i}\left[a_\circ + b\left(\frac{a_\circ}{b}\cos\theta - \frac{v^2}{b^2}\sin\theta\right)\right] - \hat{j}b\left(\frac{a_\circ}{b}\sin\theta + \frac{v^2}{b^2}\cos\theta\right)$$

$$\left|\vec{a}_P\right| = a_\circ\left(2 + 2\cos\theta + \frac{v^4}{a_\circ^2 b^2} - \frac{2v^2}{a_\circ b}\sin\theta\right)^{\frac{1}{2}}$$

$\vec{a}_P$ is a maximum at $\theta = 0$, i.e., at the top of the wheel.

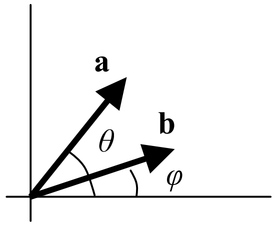
$$-2\sin\theta - \frac{2v^2}{a_\circ b}\cos\theta = 0$$

$$\theta = \tan^{-1}\left(-\frac{v^2}{a_\circ b}\right)$$

1.29 $\tilde{\boldsymbol{R}}\boldsymbol{R} = \begin{pmatrix} x & -x & 0 \\ x & x & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x & x & 0 \\ -x & x & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 2x^2 & 0 & 0 \\ 0 & 2x^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ Therefore, $x = \frac{1}{\sqrt{2}}$

The transformation represents a rotation of 45° about the z-axis (see Example 1.8.2)

1.30



(a) $\boldsymbol{a} = \hat{\boldsymbol{i}}\cos\theta + \hat{\boldsymbol{j}}\sin\theta$

$\boldsymbol{b} = \hat{\boldsymbol{i}}\cos\varphi + \hat{\boldsymbol{j}}\sin\varphi$

$\boldsymbol{a}\cdot\boldsymbol{b} = \cos(\theta-\varphi) = \left(\hat{\boldsymbol{i}}\cos\theta + \hat{\boldsymbol{j}}\sin\theta\right)\cdot\left(\hat{\boldsymbol{i}}\cos\varphi + \hat{\boldsymbol{j}}\sin\varphi\right)$

$\cos(\theta-\varphi) = \cos\theta\cos\varphi + \sin\theta\sin\varphi$

(b) $\boldsymbol{b}\times\boldsymbol{a} = \left|\hat{\boldsymbol{k}}\right|\sin(\theta-\varphi) = \left|\left(\hat{\boldsymbol{i}}\cos\theta + \hat{\boldsymbol{j}}\sin\theta\right)\times\left(\hat{\boldsymbol{i}}\cos\varphi + \hat{\boldsymbol{j}}\sin\varphi\right)\right|$

$\sin(\theta-\varphi) = \sin\theta\cos\varphi - \cos\theta\sin\varphi$
