Homework 8 – Thermodynamics of Materials

PHYS 324, Spring 2008, Longwood University

Due: March 24th

1. For large values of N the most-probable value is practically identical to the mean value. To show that this is indeed the case, calculate the probability of finding one mole of paramagnetic material with a number of $\uparrow$ states ($n_\uparrow$) which differs by:

   (a) one part in one billion from the most probable value,

   (b) one part in one-hundred billion from the most-probable value,

   (c) one part in one trillion from the most-probable value.

2. Consider a system consisting of N distinguishable atoms which may have one of three values: $\uparrow$, $\downarrow$ and $\leftrightarrow$. The measurable quantity is ξ, which equals $n_\uparrow - n_\downarrow$. The $n_\leftrightarrow$ atoms in the $\leftrightarrow$ state do contribute to ξ.

   (a) List all the possible microstates and the corresponding value of ξ for $N = 3$. (Hint: make a table similar to fig. 8.2 in *Intro to Thermophysics*.)

   (b) Make a table of ξ, $\Omega_N(\xi)$ and $f_N(\xi)$ for $N = 3$. (Hint: make a table similar to fig. 8.3 in *Intro to Thermophysics*.)

3. Consider a system consisting of N identical but distinguishable harmonic oscillators. Each oscillator has an energy of $\left(n + \frac{1}{2}\right) hv$. Since energy may be added to the system only in quanta of hv, we may consider the internal energy of the system as:

$$U = \frac{N}{2} hv + qhv$$

   where q is the number of quanta of energy in the system.

   (a) List all the possible microstates for $N = 3$ and $q = 2$.

   (b) Repeat for $N = 3$ and $q = 4$.

   (c) Do your results agree with the general result:

$$\Omega_N(q) = \frac{(N + q - 1)!}{q!\,(N-1)!}$$

4. Write a computer program (Matlab, Maple, Maxima, etc.) to calculate the probability of having $n_\uparrow$ spin $\uparrow$ atoms in a system of N spin $\frac{1}{2}$ atoms (either $\uparrow$ or $\downarrow$). The values N and n should be inputs. The program should calculate the probability three ways: (1) with no approximation, (2) using the Gaussian approximation and (3) using the Stirling approximation. Print the code and turn in with the rest of this assignment. Also, use this program to find the probability for each of the following cases:

   (a) $n_\uparrow = 10$, $N = 20$

   (b) $n_\uparrow = 5$, $N = 20$

   (c) $n_\uparrow = 75$, $N = 200$

(d) $n_\uparrow = 15$, $N = 200$

5. Use whatever means you have available (such as a computer algebra system) and the Stirling approximation to the binomial distribution to plot the following:

   (a) Plot $\frac{f_N(j)}{f_{max}}$ versus $\frac{j}{N}$ for $N = 100$, 1,000 and 10,000. Use an x-range of $\frac{j}{N} = \pm 0.3$. Comment on the result, specifically addressing the relative widths of the distributions with N.

   (b) Plot $f_N(n)$ versus n for $N = 100$, 1,000, 2,000 and 3,000. Comment on the result, specifically addressing the heights and relative widths of the distributions with N.