Homework 9 – Thermodynamics of Materials

PHYS 324, Spring 2008, Longwood University

Due: March 31st

1. The entropy (S) for a spin $\frac{1}{2}$ paramagnetic solid was found to be:

$$S = N\,k\,ln\,(2) - k\frac{U^2}{2\mu^2\mu_0^2\mathcal{H}^2 N},$$

where N is the number of particles in the system, U is the internal energy, $\mathcal{H}$ is the magnetic intensity (*intensive* variable), μ_0 is the permeability of free space and μ is the magnetic moment. Use this expression for entropy and the following to determine the equation of state for a paramagnetic solid:

$$\mathcal{M} = -T\left[\frac{\partial S}{\partial\left(\mu_0\mathcal{H}\right)}\right]_{U,\,N},$$

where $\mathcal{M}$ is the magnetization (*extensive* variable).

2. Consider a system of N non-interacting particles each with two possible energies: $E_1 = 0$ and $E_2 = \varepsilon\left(X\right)$. The second energy state has energy ε, which is a function of X (*extensive* variable). Determine the following:

   (a) the entropy,

   (b) internal energy,

   (c) Helmholtz free energy,

   (d) and heat capacity C_X as a function of temperature (T).

3. Consider a system of N non-interacting particles each with two possible energies: $E_1 = 0$ and $E_2 = \varepsilon = \frac{A}{V}$, where A is a constant and V is the volume. Determine the following:

   (a) the equation of state,

   (b) heat capacity C_Y,

   (c) Gibbs free energy,

   (d) and enthalpy as a function of temperature (T).

4. Consider a system consisting of N identical but distinguishable harmonic oscillators. Each oscillator has an energy of $\left(n + \frac{1}{2}\right)hv$. Since energy may be added to the system only in quanta of hv, we may consider the internal energy of the system as:

$$U = \frac{N}{2}hv + qhv$$

where q is the number of quanta of energy in the system.

   (a) Determine the temperature of the system as a function of N and q.

(b) Determine the internal energy of the system as a function of temperature.

Hint: You may find the following relations helpful:

$$\left(\frac{\partial}{\partial U}\right)_N = \left(\frac{\partial q}{\partial U}\right)_N \left(\frac{\partial}{\partial q}\right)_N,$$

and

$$\frac{\partial}{\partial x} ln(x!) = ln(x).$$

5. To model a semiconducting crystal we consider a lattice with N_0 sites, each of which may be occupied by either 0 or 1 electrons. There are N electrons distributed at random among the N_0 lattice sites.

(a) Show that the number of different arrangements of the N electrons among the N_0 lattice sites is given by the binomial distribution:

$$\Omega\left(N_0,\, N\right) = \frac{N_0!}{(N_0 - N)!\, N!}.$$

(b) Determine the entropy of the semiconductor crystal system.

(c) Determine the pressure. The volume is related to N_0 by $N_0 = nV$, where n is the number of sites per unit volume (which we assume is constant). Simply your result for $N \ll N_0$.