HW#5 - PHYS 324

① (a) AFTER COMPLETION OF A CYCLE, THE SYSTEM WILL RETURN TO ITS INITIAL STATE. SINCE ENTROPY IS A <u>STATE FUNCTION</u>, THE SYSTEM'S ENTROPY WILL BE UNCHANGED

TWO RESERVOIRS

$$\Delta S_{HOT} = -\frac{|Q_H|}{T_H} \qquad \Delta S_{COLD} = \frac{|Q_C|}{T_C}$$

WHERE

$$\Delta S_{TOTAL} = \Delta S_{HOT} + \Delta S_{COLD}$$

$$\boxed{\Delta S_{TOTAL} = -\frac{Q_H}{T_H} + \frac{Q_C}{T_C}} \qquad (1)$$

(b) THE EFFICIENCY AS A FUNCTION OF Q_C AND Q_H IS:

$$\eta = \frac{Q_H - Q_C}{Q_H} = 1 - \frac{Q_C}{Q_H} \qquad (2)$$

SOLVE (1) FOR Q_C YIELDS:

$$Q_C = T_C \Delta S_{TOTAL} + \frac{T_C}{T_H} Q_H \qquad (3)$$

(3) INTO (2):

$$\eta = 1 - \frac{T_C \Delta S_{TOTAL} + \frac{T_C}{T_H} Q_H}{Q_H}$$

$$\eta = 1 - \frac{T_C}{T_H} - \frac{T_C \Delta S_{TOTAL}}{Q_H} \qquad (4)$$

THE SECOND LAW REQUIRES THAT $\Delta S_{TOTAL} \geq \phi$, SO (4) IS MAXIMUM WHEN $\Delta S_{TOTAL} = \phi$ ∴

$$\boxed{\eta_{MAX} = 1 - \frac{T_C}{T_H}}$$

HW #5 - PHYS 324

(2) THE DEFINITION OF ENTROPY

$$dS = \frac{dq}{T}$$

SINCE THE TEMP IS CONSTANT

$$\Delta S = \frac{1}{T}\int dq = \frac{\Delta q}{T}$$

1st LAW IN DIFF. FORM:

$$dU = dq + dw$$

REMEMBER, WE SHOWED THAT FOR AN IDEAL GAS, THE INT ENERGY IS A FUNCTION OF TEMP. ONLY. SINCE T IS CONST

$$\Delta U(T) = \phi = dq + dw$$

$\therefore$

$$dq = -dw$$

OR

$$\Delta q = \int_{V_i}^{V_f} P\,dV = \int_{V_i}^{V_f} \frac{nRT}{V}\,dV$$

$$\Delta q(V) = nRT \ln\left(\frac{V_f}{V_i}\right)$$

$\therefore$ THE ENTROPY CHANGE IS:

$$\Delta S = nR\ln\left(\frac{V_f}{V_i}\right) \quad \text{OR} \quad \boxed{\Delta \bar{S} = R\ln\left(\frac{V_f}{V_i}\right)}$$

NOTE THAT THE RESERVOIR GAVE UP THE <u>SAME</u> AMOUNT OF HEAT @ THE SAME TEMP, SO $\Delta S_{UNIVERSE} = \phi$

$$\Delta \bar{S}_{UNIVERSE} = \Delta \bar{S}_{gas} + \Delta S_{RES} = R\ln\left(\frac{V_f}{V_i}\right) - R\ln\left(\frac{V_f}{V_i}\right) = \phi$$

FOR FREE EXPANSION, THE ENTROPY CHANGE FOR THE GAS SYSTEM IS THE <u>SAME</u> AS PISTON CASE

$$\Delta S_{gas} = R\ln\left(\frac{V_f}{V_i}\right)$$

HOWEVER, OR DUMB LITTLE DEMON ENSURES THE SURROUNDINGS STAY THE SAME. $\Delta S_{RES} = \phi$

THAT'S WHY ITS <u>IRREVERSIBLE</u> $\therefore$ $\Delta S_{UNIVERSE} = R\ln\left(\frac{V_f}{V_i}\right)$

HW#5 - PHYS 324

(3) CLAPEYRON EQUATION:

$$\frac{dT_B}{dP} = T_B \frac{\bar{V}_g - \bar{V}_l}{\Delta \bar{H}_B} \quad \text{APPROXIMATE} \quad \Delta T_B = T_B \frac{\bar{V}_g - \bar{V}_l}{\Delta \bar{H}_B} \Delta P$$

WE NEED TO FIGURE OUT MOLAR VOLUMES, THE HINT SAID TO THINK ABOUT THE DENSITIES:

$$\rho = \frac{M}{V} = \frac{n\,MW}{V} \Rightarrow V = \frac{n\,MW}{\rho} \quad \therefore \quad \bar{V} = \frac{MW}{\rho}$$

SO...

$$\bar{V}_g = \frac{MW_{H_2O}}{\rho_g} = \frac{18\ g/mol\ H_2O}{0.59\ kg/m^3} = \frac{0.018\ kg/mol}{0.59\ kg/m^3} = \boxed{0.031\ m^3/mol}$$

$$\bar{V}_l = \frac{MW_{H_2O}}{\rho_l} = \frac{18\ g/mol\ H_2O}{1000\ kg/m^3} = \frac{0.018\ kg/mol}{1000\ kg/m^3} = \boxed{1.8\times10^{-5}\ m^3/mol}$$

FOR BOILING $\Delta \bar{H}_B = 40{,}647\ J/mol$

$$\therefore \quad \Delta T_B = (373\ K)\frac{(0.031 - 1.8\times10^{-5})\ m^3/mol}{40{,}647\ J/mol}(-20{,}000\ Pa)$$

$$\boxed{\Delta T_B = -5.7\ K} \qquad \boxed{T_{B_{DENVER}} = 94.3°C}$$

SIMILARLY FOR MELTING, $\Delta \bar{H}_M = 6{,}008\ J/mol$

$$\Delta T_M = (273\ K)\frac{(1.8\times10^{-5} - 1.96\times10^{-5})\ m^3/mol}{6{,}008\ J/mol}(-20{,}000\ Pa)$$

$$\boxed{\Delta T_M = 1.45\times10^{-3}\ K}$$

THE MELTING POINT OF H_2O IN DENVER IS ACTUALLY SLIGHTLY HIGHER THAN FARMVILLE, THOUGH THE DIFF. IS TOO SMALL TO EASILY MEASURE.

4. We consider the two aluminum masses together as an isolated system. No work is done. From the first law:

$$dU = dq_{h\to c} + dq_{c\to h} = 0 \qquad (1),$$

therefore:

$$dq_{h\to c} = -dq_{c\to h} \qquad (2).$$

Considering:

$$dq(T) = C_p(T)dT \qquad (3),$$

then (2) becomes:

$$C_p^{cold}dT = -C_p^{hot}dT \qquad (4).$$

Since $C_p = n\overline{C}_p$, then (4) becomes

$$\frac{c}{c+h}\overline{C}_p^{cold}dT = -\frac{h}{c+h}\overline{C}_p^{hot}dT \qquad (5).$$

Integrate both sides:

$$\frac{c}{c+h}\int_{Tc}^{Tf}\overline{C}_p^{cold}dT = -\frac{h}{c+h}\int_{Th}^{Tf}\overline{C}_p^{hot}dT \qquad (6),$$

where:

$$T_C(\theta) = 600\left(1-\frac{\theta}{2}\right) \qquad (7),$$

$$T_H(\theta) = 600\left(1+\frac{\theta}{2}\right)$$

$$\overline{C}_p(T) = 20.7 + 0.0123\cdot T$$

I used Maxima (a free Maple clone) to evaluate the integrals and solve for $T_f(\theta,c,h)$. The Maxima worksheet is attached on the next few pages. The result is a function of *c, h* and *theta* and is shown below:

$$\mathrm{Tf}(c,h,\mathrm{theta}) := \frac{150\sqrt{(3844h^2+7688ch+3844c^2)\mathrm{theta}^2+(58156h^2-58156c^2)\mathrm{theta}+219961h^2+439922ch+219961c^2}-51750h+(-51750)c}{31h+31c}$$

In order to plot the change in entropy as a function of θ, we can evaluate similar integrals to those in (6). Since:

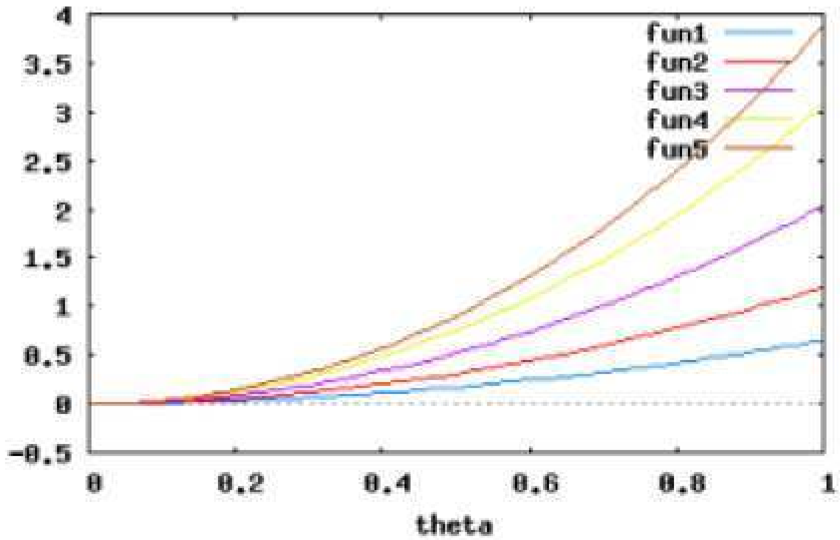
$$dS(T) = \frac{\delta q}{T} = \frac{\delta q^{cold}}{T} + \frac{\delta q^{hot}}{T} \qquad (8),$$

or,

$$\Delta\overline{S}(c,h,\theta) = \frac{c}{c+h}\int_{Tc}^{Tf}\frac{\overline{C}_p^{cold}dT}{T} + \frac{h}{c+h}\int_{Th}^{Tf}\frac{\overline{C}_p^{hot}dT}{T} \qquad (9).$$

The integrals in (9) get extremely messy (as you will see from the Maxima worksheet). We need to evaluate this expression using the expression for $Tf(c,h,\theta)$ for various values of *c* and *h* and then plot ΔS as a function of θ. I won't cut and paste the expression here, since it is rather large and messy.

I set $c = 1$ and $h = 16, 8, 4, 2$, and 1 and generated five different functions $\Delta\bar{S}(\theta)$. I then plotted all five functions. They are shown below where the top curve corresponds to $c/h = 1/16$ and the bottom curve corresponds to $c/h = 1$.



```
/*
wxMaxima 0.7.4 http://wxmaxima.sourceforge.net
Maxima 5.14.0 http://maxima.sourceforge.net
Using Lisp GNU Common Lisp (GCL) GCL 2.6.8 (aka GCL)
Distributed under the GNU Public License. See the file COPYING.
Dedicated to the memory of William Schelter.
The function bug_report() provides bug reporting information.
```

(%i1) `Cp(T):=20.7+T*(12.4e-3);`

(%o1) $\mathrm{Cp}(T) := 20.7 + T\ 0.0124$

(%i2) `Tc(theta):=600*(1-theta/2);`

(%o2) $\mathrm{Tc}(\mathrm{theta}) := 600\left(1 - \frac{\mathrm{theta}}{2}\right)$

(%i3) `Th(theta):=600*(1+theta/2);`

(%o3) $\mathrm{Th}(\mathrm{theta}) := 600\left(1 + \frac{\mathrm{theta}}{2}\right)$

(%i4) `Qc(Tf,theta,c,h):=(c/(c+h))*integrate(Cp(T),T,Tc(theta),Tf);`

(%o4) $\mathrm{Qc}(Tf, \mathrm{theta}, c, h) := \frac{c}{c+h}\ \mathrm{integrate}(\mathrm{Cp}(T), T, \mathrm{Tc}(\mathrm{theta}), Tf)$

(%i5) `Qh(Tf,theta,c,h):=(h/(c+h))*integrate(Cp(T),T,Th(theta),Tf);`

(%o5) $\mathrm{Qh}(Tf, \mathrm{theta}, c, h) := \frac{h}{c+h}\ \mathrm{integrate}(\mathrm{Cp}(T), T, \mathrm{Th}(\mathrm{theta}), Tf)$

(%i6) `solve([Qc(Tf,theta,c,h)=-Qh(Tf,theta,c,h)],Tf);`

```
`rat' replaced 20.7 by 207/10 = 20.7
`rat' replaced 0.0124 by 31/2500 = 0.0124
`rat' replaced 41.4 by 207/5 = 41.4
`rat' replaced 12420.0 by 12420/1 = 12420.0
`rat' replaced 2232.000000000001 by 2232/1 = 2232.0
`rat' replaced 20.7 by 207/10 = 20.7
`rat' replaced 0.0062 by 31/5000 = 0.0062
`rat' replaced 20.7 by 207/10 = 20.7
`rat' replaced 0.0124 by 31/2500 = 0.0124
`rat' replaced 41.4 by 207/5 = 41.4
`rat' replaced 12420.0 by 12420/1 = 12420.0
`rat' replaced 2232.000000000001 by 2232/1 = 2232.0
`rat' replaced 20.7 by 207/10 = 20.7
```

`rat' replaced 0.0062 by 31/5000 = 0.0062

(%o6) $[Tf = -(150\ sqrt((3844 h^2 + 7688 c h + 3844 c^2)\ theta^2 + (58156 h^2 - 58156 c^2)\ theta + 219961\ h^2 + 439922\ c\ h + 219961\ c^2) + 51750\ h + 51750\ c) / (31\ h + 31\ c), Tf = (150\ sqrt((3844 h^2 + 7688 c h + 3844 c^2)\ theta^2 + (58156 h^2 - 58156 c^2)\ theta + 219961\ h^2 + 439922\ c\ h + 219961\ c^2) - 51750\ h - 51750\ c) / (31\ h + 31\ c)]$

(%i7)

```
Tf(c,h,theta):=(150*sqrt((3844*h^2+7688*c*h+3844*c^2)*theta^2+(58156*h^2-58156*c
 +219961*h^2+439922*c*h+219961*c^2)-51750*h-51750*c)/(31*h+31*c);
```

(%o7) $\mathrm{Tf}(c, h, theta) := (150\ sqrt((3844 h^2 + 7688 c h + 3844 c^2)\ theta^2 + (58156 h^2 - 58156 c^2)\ theta + 219961\ h^2 + 439922\ c\ h + 219961\ c^2) - 51750\ h + (-51750)\ c) / (31\ h + 31\ c)$

(%i8)

```
Sc(theta,c,h):=(c/(c+h))*integrate(Cp(T)/T,T,Tc(theta),Tf(c,h,theta));
```

(%o8) $\mathrm{Sc}(theta, c, h) := \frac{c}{c+h}\ \mathrm{integrate}\left(\frac{\mathrm{Cp}(T)}{T}, T, \mathrm{Tc}(theta), \mathrm{Tf}(c, h, theta)\right)$

(%i9)

```
Sh(theta,c,h):=(h/(c+h))*integrate(Cp(T)/T,T,Th(theta),Tf(c,h,theta));
```

(%o9) $\mathrm{Sh}(theta, c, h) := \frac{h}{c+h}\ \mathrm{integrate}\left(\frac{\mathrm{Cp}(T)}{T}, T, \mathrm{Th}(theta), \mathrm{Tf}(c, h, theta)\right)$

(%i10) `S(theta,c,h):=Sc(theta,c,h)+Sh(theta,c,h);`

(%o10) $\mathrm{S}(theta, c, h) := \mathrm{Sc}(theta, c, h) + \mathrm{Sh}(theta, c, h)$

(%i11) `S(theta,c,h);`

Is $\left(3844 h^2 + 7688 c h + 3844 c^2\right) theta^2 + \left(58156 h^2 - 58156 c^2\right) theta + 219961\ h^2 + 439922\ c\ h + 219961\ c^2$ positive, negative, or zero? positive;

Is $(h + c)\ (150\ sqrt(\left(3844 h^2 + 7688 c h + 3844 c^2\right) theta^2 + \left(58156 h^2 - 58156 c^2\right) theta + 219961\ h^2 + 439922\ c\ h + 219961\ c^2) + (9300 h + 9300 c)\ theta - 70350\ h - 70350\ c)$ positive, negative, or zero? positive;

Is $(h+c)\,(150\,\mathrm{sqrt}((3844\,h^2+7688\,c\,h+3844\,c^2)\,\mathrm{theta}^2+(58156\,h^2-58156\,c^2)\,\mathrm{theta}+219961\,h^2+439922\,c\,h+219961\,c^2)-51750\,h-51750\,c)$ *positive, negative, or zero?* positive;

Is $\mathrm{theta}-2$ *positive, negative, or zero?* negative;

`rat' replaced 20.7 by 207/10 = 20.7
`rat' replaced -0.0124 by -31/2500 = -0.0124
`rat' replaced 20.7 by 207/10 = 20.7
`rat' replaced 0.0124 by 31/2500 = 0.0124
`rat' replaced 20.7 by 207/10 = 20.7
`rat' replaced 0.0124 by 31/2500 = 0.0124
`rat' replaced 20.7 by 207/10 = 20.7
`rat' replaced 0.0124 by 31/2500 = 0.0124

Is $(h+c)\,(150\,\mathrm{sqrt}((3844\,h^2+7688\,c\,h+3844\,c^2)\,\mathrm{theta}^2+(58156\,h^2-58156\,c^2)\,\mathrm{theta}+219961\,h^2+439922\,c\,h+219961\,c^2)+(-9300\,h-9300\,c)\,\mathrm{theta}-70350\,h-70350\,c)$ *positive, negative, or zero?* positive;

Is $\mathrm{theta}+2$ *positive, negative, or zero?* positive;

`rat' replaced 20.7 by 207/10 = 20.7
`rat' replaced -0.0124 by -31/2500 = -0.0124
`rat' replaced 20.7 by 207/10 = 20.7
`rat' replaced 0.0124 by 31/2500 = 0.0124
`rat' replaced 20.7 by 207/10 = 20.7
`rat' replaced 0.0124 by 31/2500 = 0.0124
`rat' replaced 20.7 by 207/10 = 20.7
`rat' replaced 0.0124 by 31/2500 = 0.0124

(%o11) $(h\,(((1035\,h+1035\,c)\,\log((150\,\mathrm{sqrt}((3844\,h^2+7688\,c\,h+3844\,c^2)\,\mathrm{theta}^2+(58156\,h^2-58156\,c^2)\,\mathrm{theta}+219961\,h^2+439922\,c\,h+219961\,c^2)-51750\,h-51750\,c)/(31\,h+31\,c))+3\,\mathrm{sqrt}((3844\,h^2+7688\,c\,h+3844\,c^2)\,\mathrm{theta}^2+(58156\,h^2-58156\,c^2)\,\mathrm{theta}+219961\,h^2+439922\,c\,h+219961\,c^2)-1035\,h-1035\,c)/(50\,h+50\,c)-\frac{1035\,\log(300\,\mathrm{theta}+600)+186\,\mathrm{theta}+372}{50}))/(h+c)+(c\,(((1035\,h+1035\,c)\,\log((150\,\mathrm{sqrt}((3844\,h^2+7688\,c\,h+3844\,c^2)\,\mathrm{theta}^2+(58156\,h^2-58156\,c^2)\,\mathrm{theta}+219961\,h^2+439922\,c\,h+219961\,c^2)-51750\,h-51750\,c)/(31\,h+31\,c))+3\,\mathrm{sqrt}((3844\,h^2+7688\,c\,h+3844\,c^2)\,\mathrm{theta}^2+(58156\,h^2-58156\,c^2)\,\mathrm{theta}+219961\,h^2+439922\,c\,h+219961\,c^2)-$

1035 h - 1035 c) / (50 h + 50 c) + $\frac{186\ theta - 1035\ log(600 - 300\ theta) - 372}{50}$)) / (h + c)

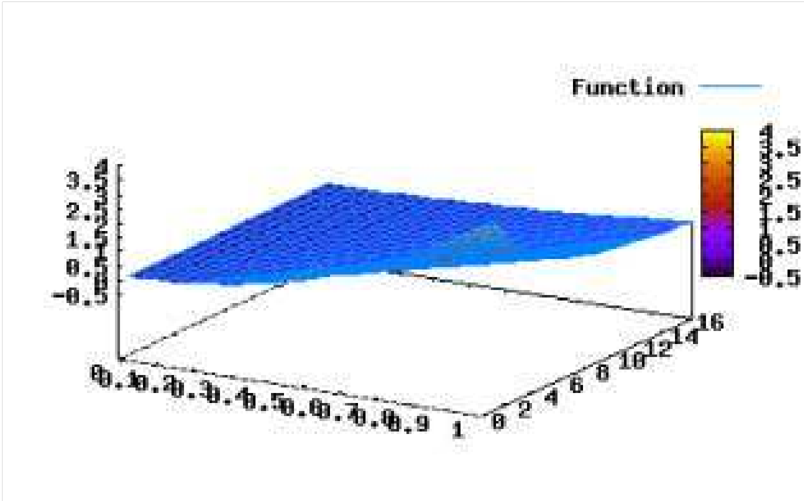
```
(%i12) c:1$
```

```
(%i13)
(h*(((1035*h+1035*c)*log((150*sqrt((3844*h^2+7688*c*h+3844*c^2)*theta^2+(58156*h
 -58156*c^2)*theta+219961*h^2+439922*c*h+219961*c^2)-51750*h-51750*c)/(31*h+31*c
 +3*sqrt((3844*h^2+7688*c*h+3844*c^2)*theta^2+(58156*h^2-58156*c^2)*theta+219961
 +439922*c*h+219961*c^2)-1035*h-1035*c)/(50*h+50*c)-(1035*log(300*theta+600)+186
 +372)/50))/(h+c)+(c*(((1035*h+1035*c)*log((150*sqrt((3844*h^2+7688*c*h+3844*c^2
 +(58156*h^2-58156*c^2)*theta+219961*h^2+439922*c*h+219961*c^2)-51750*h-51750*c)
 +31*c))+3*sqrt((3844*h^2+7688*c*h+3844*c^2)*theta^2+(58156*h^2-58156*c^2)*theta
 +439922*c*h+219961*c^2)-1035*h-1035*c)/(50*h+50*c)+(186*theta-1035*log(600-300*
 -372)/50))/(h+c);
```

(%o13) (h (((1035 h + 1035) log ((150 $\sqrt{(3844\,h^2 + 7688\,h + 3844)theta^2 + (58156\,h^2 - 58156)theta + 219961\,h^2 + 439922\,h + 219961}$ - 51750 h - 51750) / (31 h + 31)) + 3 $\sqrt{(3844\,h^2 + 7688\,h + 3844)theta^2 + (58156\,h^2 - 58156)theta + 219961\,h^2 + 439922\,h + 219961}$ - 1035 h - 1035) / (50 h + 50) - $\frac{1035\ log(300\ theta + 600) + 186\ theta + 372}{50}$)) / (h + 1) + (((1035 h + 1035) log ((150 $\sqrt{(3844\,h^2 + 7688\,h + 3844)theta^2 + (58156\,h^2 - 58156)theta + 219961\,h^2 + 439922\,h + 219961}$ - 51750 h - 51750) / (31 h + 31)) + 3 $\sqrt{(3844\,h^2 + 7688\,h + 3844)theta^2 + (58156\,h^2 - 58156)theta + 219961\,h^2 + 439922\,h + 219961}$ - 1035 h - 1035) / (50 h + 50) + $\frac{186\ theta - 1035\ log(600 - 300\ theta) - 372}{50}$) / (h + 1)
)

```
(%i16) wxplot3d(%o13, [theta,0,1], [h,1,16])$
```

(%t16)



(%i21)

```
h:16$
```

(%i22)

```
(h*(((1035*h+1035)*log((150*sqrt((3844*h^2+7688*h+3844)*theta^2+(58156*h^2-5815
 +219961*h^2+439922*h+219961)-51750*h-51750)/(31*h+31))+3*sqrt((3844*h^2+7688*h+
 +(58156*h^2-58156)*theta+219961*h^2+439922*h+219961)-1035*h-1035)/(50*h+50)-(10
 +600)+186*theta+372)/50))/(h+1)+(((1035*h+1035)*log((150*sqrt((3844*h^2+7688*h+
 +(58156*h^2-58156)*theta+219961*h^2+439922*h+219961)-51750*h-51750)/(31*h+31))+
 +7688*h+3844)*theta^2+(58156*h^2-58156)*theta+219961*h^2+439922*h+219961)-1035*
 +50)+(186*theta-1035*log(600-300*theta)-372)/50)/(h+1);
```

(%o22)

$$\left(16\left(\left(17595\log\left(\frac{150\sqrt{1110916\,theta^2+14829780\,theta+63568729}-879750}{527}\right)+3\sqrt{1110916\,theta^2+14829780\,theta+63568729}-17595\right)/850-\frac{1035\log(300\,theta+600)+186\,theta+372}{50}\right)\right)/17+\left(\left(17595\log\left(\frac{150\sqrt{1110916\,theta^2+14829780\,theta+63568729}-879750}{527}\right)+3\sqrt{1110916\,theta^2+14829780\,theta+63568729}-17595\right)/850+\frac{186\,theta-1035\log(600-300\,theta)-372}{50}\right)/17$$

(%i23)

```
h:8$
```

(%i24)

```
(h*(((1035*h+1035)*log((150*sqrt((3844*h^2+7688*h+3844)*theta^2+(58156*h^2-58156
 +219961*h^2+439922*h+219961)-51750*h-51750)/(31*h+31))+3*sqrt((3844*h^2+7688*h+
 +(58156*h^2-58156)*theta+219961*h^2+439922*h+219961)-1035*h-1035)/(50*h+50)-(10
 +600)+186*theta+372)/50))/(h+1)+(((1035*h+1035)*log((150*sqrt((3844*h^2+7688*h+
 +(58156*h^2-58156)*theta+219961*h^2+439922*h+219961)-51750*h-51750)/(31*h+31))+
 +7688*h+3844)*theta^2+(58156*h^2-58156)*theta+219961*h^2+439922*h+219961)-1035*
 +50)+(186*theta-1035*log(600-300*theta)-372)/50)/(h+1);
```

(%o24)
$$\left(8\left(\left(9315\log\left(\frac{150\sqrt{311364\,theta^{2}+3663828\,theta+17816841}-465750}{279}\right)+3\sqrt{311364\,theta^{2}+3663828\,theta+17816841}-9315\right)/450-\frac{1035\log(300\,theta+600)+186\,theta+372}{50}\right)\right)/9+\left(\left(9315\log\left(\frac{150\sqrt{311364\,theta^{2}+3663828\,theta+17816841}-465750}{279}\right)+3\sqrt{311364\,theta^{2}+3663828\,theta+17816841}-9315\right)/450+\frac{186\,theta-1035\log(600-300\,theta)-372}{50}\right)/9$$

(%i25)
```
h:4$
```

(%i26)

```
(h*(((1035*h+1035)*log((150*sqrt((3844*h^2+7688*h+3844)*theta^2+(58156*h^2-58156
 +219961*h^2+439922*h+219961)-51750*h-51750)/(31*h+31))+3*sqrt((3844*h^2+7688*h+
 +(58156*h^2-58156)*theta+219961*h^2+439922*h+219961)-1035*h-1035)/(50*h+50)-(10
 +600)+186*theta+372)/50))/(h+1)+(((1035*h+1035)*log((150*sqrt((3844*h^2+7688*h+
 +(58156*h^2-58156)*theta+219961*h^2+439922*h+219961)-51750*h-51750)/(31*h+31))+
 +7688*h+3844)*theta^2+(58156*h^2-58156)*theta+219961*h^2+439922*h+219961)-1035*
 +50)+(186*theta-1035*log(600-300*theta)-372)/50)/(h+1);
```

(%o26)
$$\left(4\left(\left(5175\log\left(\frac{150\sqrt{96100\,theta^{2}+872340\,theta+5499025}-258750}{155}\right)+3\sqrt{96100\,theta^{2}+872340\,theta+5499025}-5175\right)/250-\frac{1035\log(300\,theta+600)+186\,theta+372}{50}\right)\right)/5+\left(\left(5175\log\left(\frac{150\sqrt{96100\,theta^{2}+872340\,theta+5499025}-258750}{155}\right)+3\sqrt{96100\,theta^{2}+872340\,theta+5499025}-5175\right)/250+\frac{186\,theta-1035\log(600-300\,theta)-372}{50}\right)/5$$

(%i27) h:2$

(%i28)

```
(h*(((1035*h+1035)*log((150*sqrt((3844*h^2+7688*h+3844)*theta^2+(58156*h^2-5815
 +219961*h^2+439922*h+219961)-51750*h-51750)/(31*h+31))+3*sqrt((3844*h^2+7688*h+
 +(58156*h^2-58156)*theta+219961*h^2+439922*h+219961)-1035*h-1035)/(50*h+50)-(1C
 +600)+186*theta+372)/50))/(h+1)+(((1035*h+1035)*log((150*sqrt((3844*h^2+7688*h+
 +(58156*h^2-58156)*theta+219961*h^2+439922*h+219961)-51750*h-51750)/(31*h+31))+
 +7688*h+3844)*theta^2+(58156*h^2-58156)*theta+219961*h^2+439922*h+219961)-1035*
 +50)+(186*theta-1035*log(600-300*theta)-372)/50)/(h+1);
```

(%o28) $\left(2\left(\left(3105\log\left(\frac{150\sqrt{34596\,theta^2+174468\,theta+1979649}-155250}{93}\right)+3\sqrt{34596\,theta^2+174468\,theta+1979649}-3105\right)/150-\frac{1035\log(300\,theta+600)+186\,theta+372}{50}\right)\right)/3+\left(\left(3105\log\left(\frac{150\sqrt{34596\,theta^2+174468\,theta+1979649}-155250}{93}\right)+3\sqrt{34596\,theta^2+174468\,theta+1979649}-3105\right)/150+\frac{186\,theta-1035\log(600-300\,theta)-372}{50}\right)/3$

(%i29) h:1$

(%i30)

```
(h*(((1035*h+1035)*log((150*sqrt((3844*h^2+7688*h+3844)*theta^2+(58156*h^2-5815
 +219961*h^2+439922*h+219961)-51750*h-51750)/(31*h+31))+3*sqrt((3844*h^2+7688*h+
 +(58156*h^2-58156)*theta+219961*h^2+439922*h+219961)-1035*h-1035)/(50*h+50)-(1C
 +600)+186*theta+372)/50))/(h+1)+(((1035*h+1035)*log((150*sqrt((3844*h^2+7688*h+
 +(58156*h^2-58156)*theta+219961*h^2+439922*h+219961)-51750*h-51750)/(31*h+31))+
 +7688*h+3844)*theta^2+(58156*h^2-58156)*theta+219961*h^2+439922*h+219961)-1035*
 +50)+(186*theta-1035*log(600-300*theta)-372)/50)/(h+1);
```

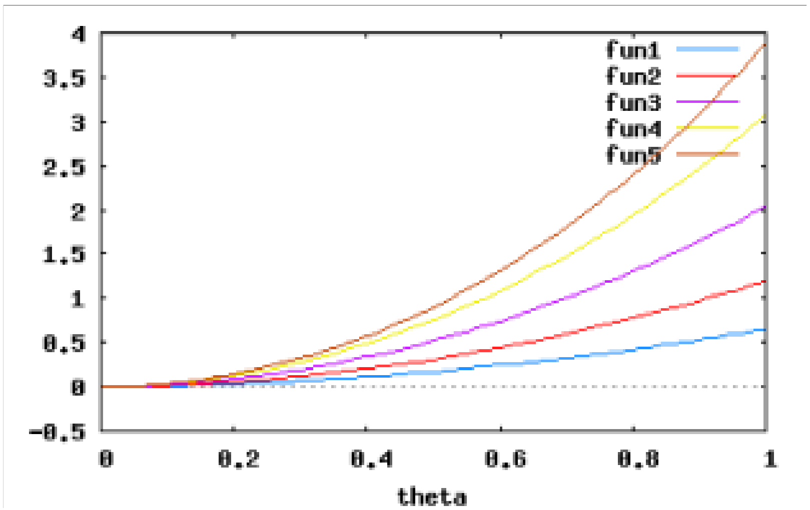
(%o30) $\left(\frac{2070\log\left(\frac{150\sqrt{15376\,theta^2+879844}-103500}{62}\right)+3\sqrt{15376\,theta^2+879844}-2070}{100}-\frac{1035\log(300\,theta+600)+186\,theta+372}{50}\right)/2+($

$$\frac{2070\log\left(\frac{150\sqrt{15376\,theta^{2}+879844}-103500}{62}\right)+3\sqrt{15376\,theta^{2}+879844}-2070}{100}+$$

$$\frac{186\,theta-1035\log(600-300\,theta)-372}{50})\,/\,2$$

(%i31) wxplot2d([%o22,%o24,%o26,%o28,%o30], [theta,0,1])$

(%t31)



(%i32)